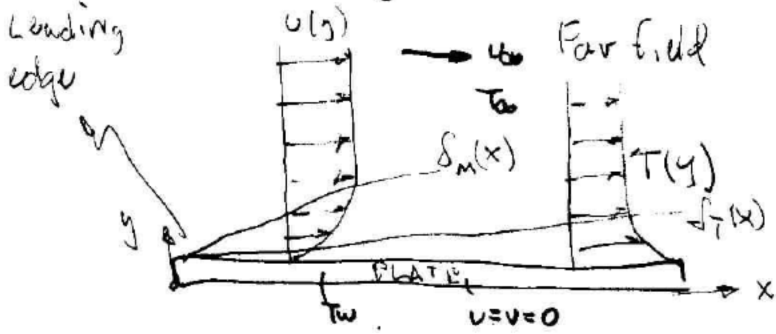
Cooling of Flat Plate



$\bar{V} = u(x,y)\hat{i} + v(x,y)\hat{j}$

$T = T(x,y)$

$u(0,y) = U_\infty$, $T(0,y) = T_\infty$ } leading edge, $\eta = \infty$

$u(x,0) = 0$, $v(x,0) = 0$, $T(x,0) = T_w$ } @ wall $\eta = 0$

$T(x,\infty) = T_\infty$, $u(x,\infty) = U_\infty$ } far field $\eta = \infty$

Mass cons. $u_x + v_y = 0$

u Mom. cons. $uu_x + vu_y = \nu u_{yy}$

Energy Cons. $uT_x + vT_y = \alpha T_{yy}$

} So called Boundary layer Equations.

Use the transform $u = U_\infty F'(\eta)$

$$v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta F'(\eta) - F(\eta)\right]$$

where $\eta = \dfrac{y}{\sqrt{\nu x / U_\infty}}$

and for temperature $G(\eta, Pr) = \dfrac{T - T_\infty}{T_w - T_\infty}$

$Pr^* = \qquad = \nu/\alpha$

$= \dfrac{\text{momentum diffusion}}{\text{thermal diffusion}}$

$\Rightarrow$ relative thickness of thermal & momentum B.L.s.

Equations become

$$\boxed{\begin{aligned} &F''' + \tfrac{1}{2}FF'' = 0 \\ &F(0) = 0 \\ &F'(0) = 0 \\ &F'(\infty) = 1 \end{aligned}}$$

Independent of G!

$$\boxed{\begin{aligned} &G'' + \frac{Pr\,F G'}{2} = 0 \\ &G(0) = 1 \\ &G(\infty) = 0 \end{aligned}}$$

$f(0) = 0$

$f'(0) = 0$

$f''(0) = 0.332057$

$x = 0$ to 10

$h = \Delta x = 0.1$

Boundary Value Problems.

Must use the technique of 'shooting' (3)

For F problem use $F(0)=0$

$F'(0)=0$

$F''(0)=\text{guess}_i$

Solve problem with guess_0 and guess_1. Interpolate to find guess_2 that will force $F'(\infty)\to 1$.

Iterate! By Hand or False-Position

Once $F(\eta)$ is solved, then solve G problem.

Pick $G'(0) = \text{guess}_0$ & guess_1. Run the problem to iterate to $G'(0)$ that gives $G(\infty)=0$.

Use $F(0)=0$ $\quad G(0)=1$

$F'(0)=0$ $\quad G'(0)=-.576689$

$F''(0)=.332057$

$Pr=5$

$x=0$ to 10

$\eta_m=4.91$

$\eta_t=2.75$

To use R-K

$F = y_1$

$F' = y_1' = y_2$

$F'' = y_2' = y_3$

$F''' = y_3' = -\frac{1}{2} FF''$

$G = y_4$

$G' = y_4' = y_5$

$G'' = y_5' = -\frac{1}{2} P_r F G'$

$y_1' = y_2$	$y_1(0) = 0$
$y_2' = y_3$	$y_2(0) = 0$
$y_3' = -\frac{1}{2} y_1 y_3$	$y_3(0) = \text{guess}$
$y_4' = y_5$	$y_4(0) = 1$
$y_5' = -\frac{1}{2} P_r y_1 y_5$	$y_5(0) = \text{guess}$

Run through 4th order R-K.

guess 1 | guess 2

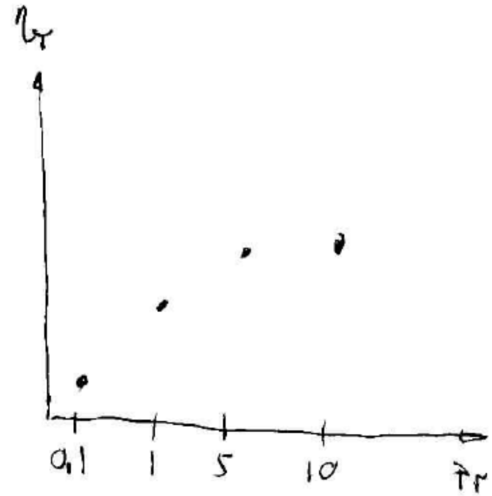
$x = \eta$	F	F'	F''	G	G'
0	0	0	3.32057-1	1.0	-5.76689-1
0.1	1.66029-3	3.32055-2	3.32048-1	9.42333-1	-5.76609-1
0.2	6.64101-3	6.64078-2	3.31984-1	8.84694-1	-5.76051-1
η_m :		0.990000			
η_T :				0.310000	
9.8	8.07921	1	1.91607-8	-1.82232-16	-9.97149-32
9.9	8.17921	1	1.27627-8	-1.82232-16	-5.43710-32
10.0	8.27921	1	8.45581-9	-1.82232-16	-1.21701-32

Asympt. to 1

asympt. to 0

- ONLY ONE SOLUTION
- Does not depend on $G(\eta)$ solution

- CHANGES WITH Pr
- Depends on $F(\eta)$ solution



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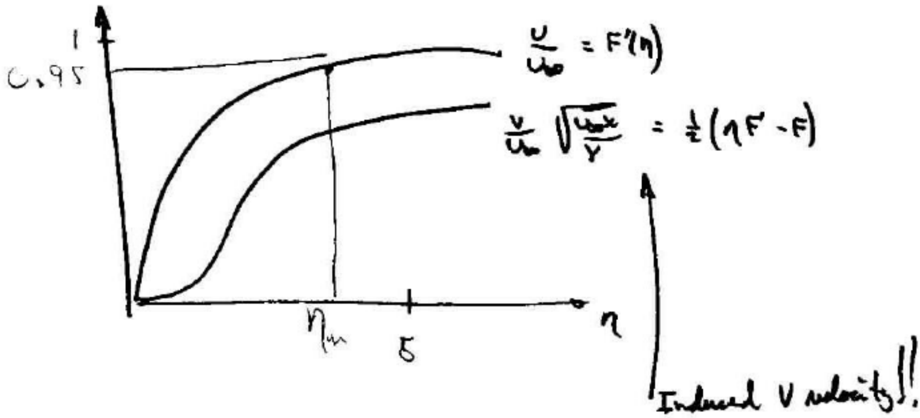
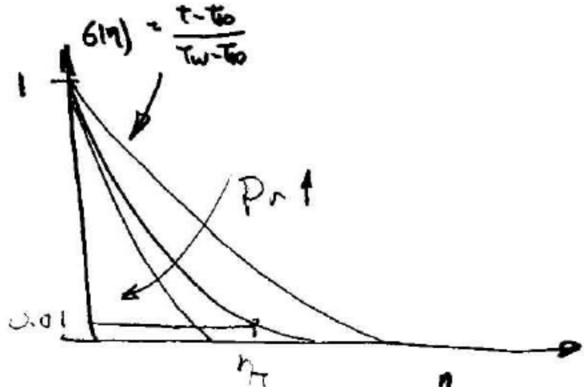
END

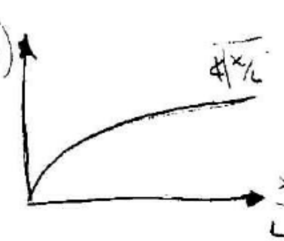
Velocity Profile



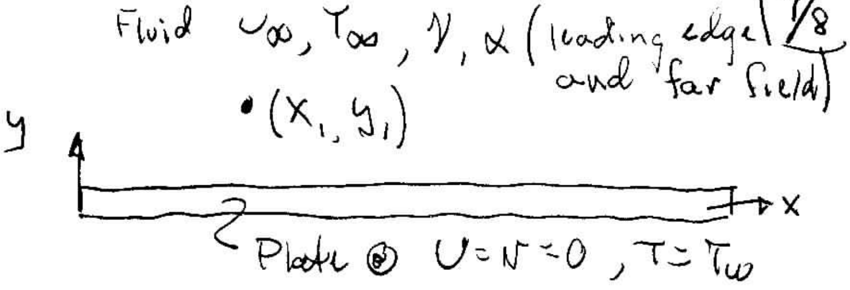


$K = \eta_T - \eta_m$

Do this for H.W.



Ex) Sort of...



Find u, v, T @ (x_1, y_1)

Calc $\eta_1 = \dfrac{y_1}{\sqrt{\nu x_1 / U_\infty}} = \ldots$ (known value)

$Pr_1 = \nu / \alpha = \ldots$ (known value)

Go to

η	F	F'	F''	G	G'
η_1	F_1	F_1'	F_1''	G_1	G_1'

table for your Pr_1 value

Grab the corresponding $F_1, F_1', F_1'', G_1, G_1'$ values

Then $G_1 = \dfrac{T_1 - T_\infty}{T_w - T_\infty} \Rightarrow T_1 = \ldots$

$u_1 = U_\infty F'(\eta_1) = \ldots$

$v_1 = \dfrac{1}{2}\sqrt{\dfrac{\nu U_\infty}{x_1}} \left[\eta_1 F'(\eta_1) - F(\eta_1) \right] = \ldots$

You now know u_1, v_1, T_1 @ (x_1, y_1) $\Bigg\}$ Can now calculate

$\tau_w = \nu \dfrac{\partial u}{\partial y}\Big|_{y=0} \sim F''\Big|_{\eta_1}$

$q'' = -k \dfrac{\partial T}{\partial y}\Big|_{y=0} \sim G'\Big|_{\eta}$

Specifically $\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu U_\infty \sqrt{\frac{U_\infty}{\nu x_1}} \left. F'' \right|_{\eta=0} = 0.332 \sqrt{\frac{\rho \mu U_\infty}{x_1}}$

$$= 0.332 \frac{\rho U_\infty^2}{\sqrt{Re_{x_1}}}$$

and $\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s) \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (T_\infty - T_s) \left.\frac{d\theta}{d\eta}\right|_{\eta=0} \left.\frac{\partial \eta}{\partial y}\right|_{y=0}$

$$= 0.332 \, Pr^{1/3} (T_\infty - T_s) \sqrt{\frac{U_\infty}{\nu x_1}}$$

so that

$$h_x = \frac{\left. q'' \right|_{surf}}{(T_s - T_\infty)} = \frac{-k\left(\partial T / \partial y\right)_{y=0}}{(T_s - T_\infty)} = 0.332 \, Pr^{1/3} k \sqrt{\frac{U_\infty}{\nu x_1}}$$

where $Re_{x_1} = \frac{U_\infty x_1}{\nu}$

(*) $C_{f,x_1} = \frac{\tau_w}{\left(\frac{1}{2} \rho U_\infty^2\right)} = 0.664 / \sqrt{Re_{x_1}}$